Modelling of atomisation processes in fuel injection systems

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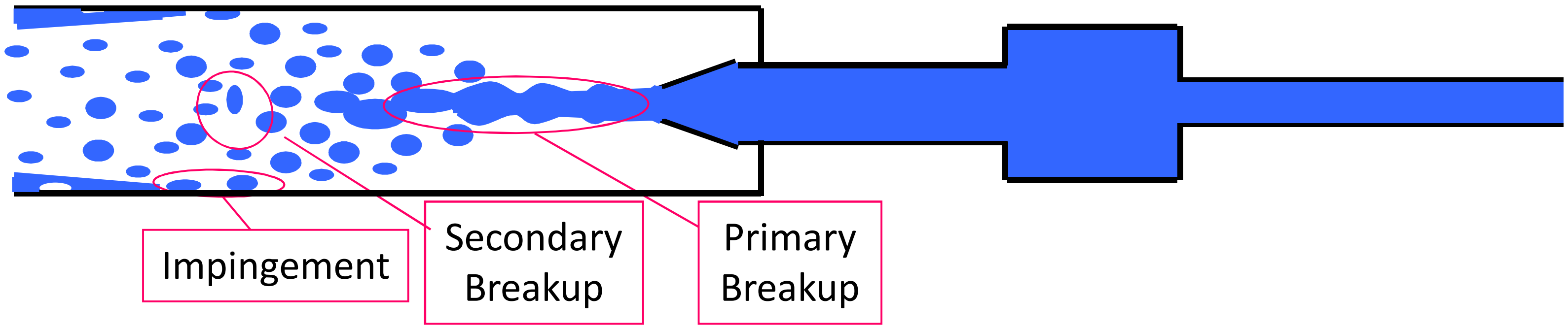
Atomisation processes and Sprays

Atomisation processes:

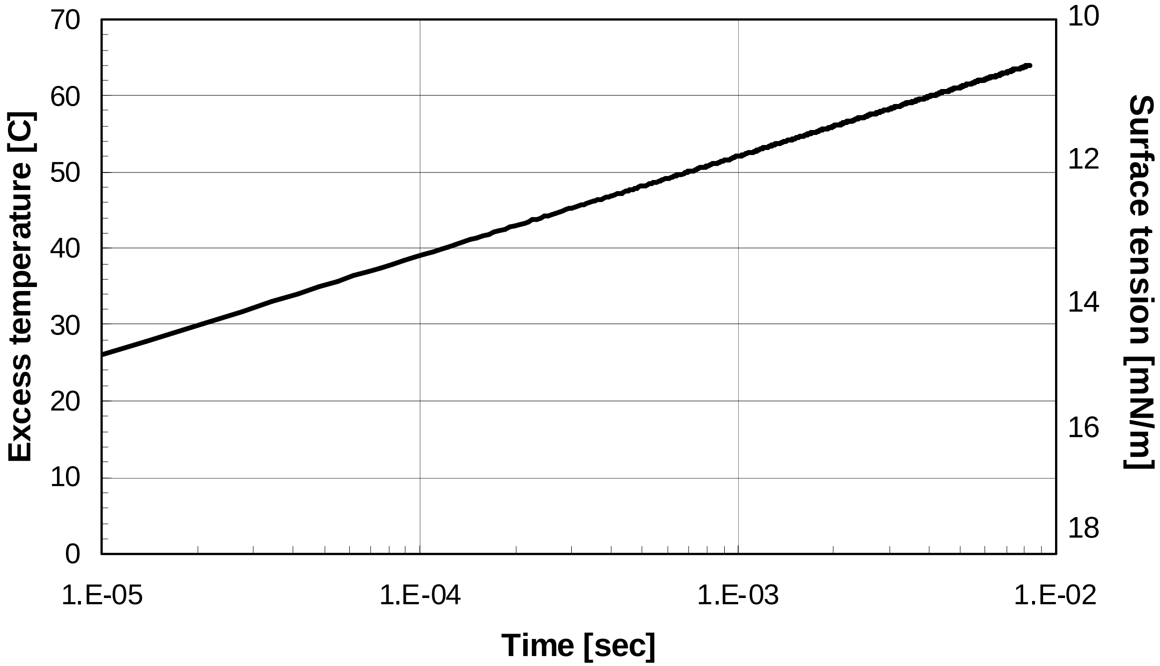
- Primary breakup: jet breakup
- Secondary breakup: droplet breakup
- Spray interaction: droplet impingement



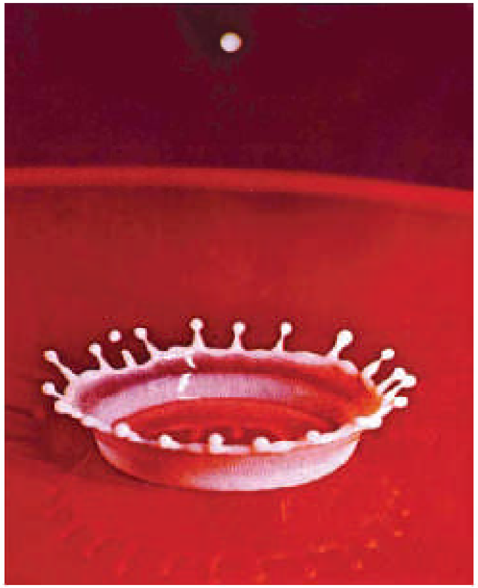
Transient parameters:

- Liquid velocity
- Physical Properties: Surface Tension (with temperature)

Variation of Surface Tension
(Semi-finite liquid model. dT~1000K)



Droplet Impingement



Important parameters:

-Droplet diameter and velocity
-Wet/Dry surface
-Surface tension
-Viscosity

Droplet Impingement

<u>Overall energy calculation</u>

$$\Delta E_{kinetic} + \Delta E_{surface} + \Delta E_{viscous} = 0$$

$$\Downarrow$$

$$D_{final} = f\left(D_{initial}, v_{impact}, properties\right)$$

1. Simple
2. Can account for partially wetted surfaces

1. No dynamic behavior
2. No splashing prediction

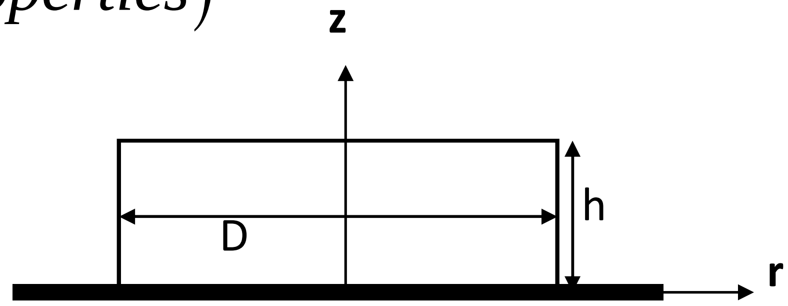
Droplet Impingement

Temporal energy calculation

$$d\left(E_{kinetic} + E_{surface} + E_{viscous}\right) = 0$$

$$\Downarrow$$

$$h(t) = f\left(t, D_{initial}, v_{impact}, properties\right)$$



1. Simple
2. Can account for partially wetted surfaces
3. Dynamic behavior

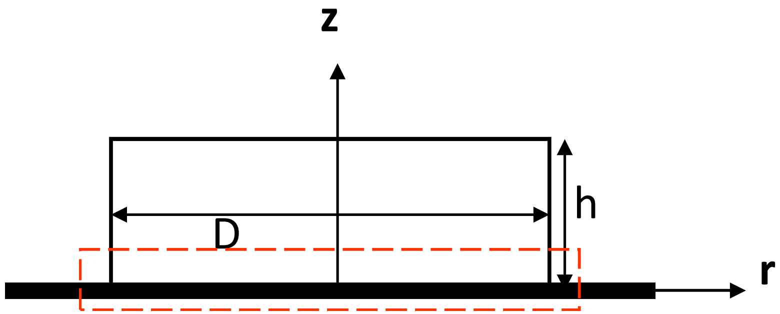
1. Restricted to highly simplified geometries
2. No splashing prediction

Droplet Impingement

Temporal energy calculation

$$E_{kinetic} + E_{surface} + E_{viscous} = E$$

Potential flow: $v_r \approx -\frac{1}{2}\frac{r}{\bar{h}}\dot{h}$ $\quad v_z \approx \frac{z}{\bar{h}}\dot{h}$



$$E_{kinetic} = \frac{1}{2}\rho\iiint_{\hat{V}} v^2 d\hat{V} = \frac{1}{2}m\left(\frac{1}{\bar{h}}\int_0^h \frac{z^2}{\bar{h}^2}\dot{h}^2 dz + \frac{1}{\pi R^2 \bar{h}}\int_0^h \frac{r^2}{\bar{h}^2}\dot{h}^2 \cdot 2\pi r \bar{h}\, dr\right) = \left(\frac{m}{6} + \frac{m^2}{16\pi\rho}\frac{1}{h^3}\right)\dot{h}^2$$

$$E_{surface} = \pi R^2\left(\sigma_{\lg} + \sigma_{sl} - \sigma_{sg}\right) = \frac{m}{\rho}\left(\sigma_{\lg} + \sigma_{sl} - \sigma_{sg}\right)\frac{1}{\bar{h}}$$

$$E_{viscous} = C\pi R^3 \rho V^2 \frac{1}{\sqrt{\mathrm{Re}}} = \frac{4}{3}CmV^2\frac{1}{\sqrt{\mathrm{Re}}}$$

Non-dimensionalisation:

$$\tilde{h} \equiv \frac{h}{\frac{1}{2}D} \quad \tilde{t} \equiv \frac{t}{\frac{1}{2}D/V} \quad \tilde{R} \equiv \frac{R}{\frac{1}{2}D} \quad \tilde{r} \equiv \frac{r}{R}$$

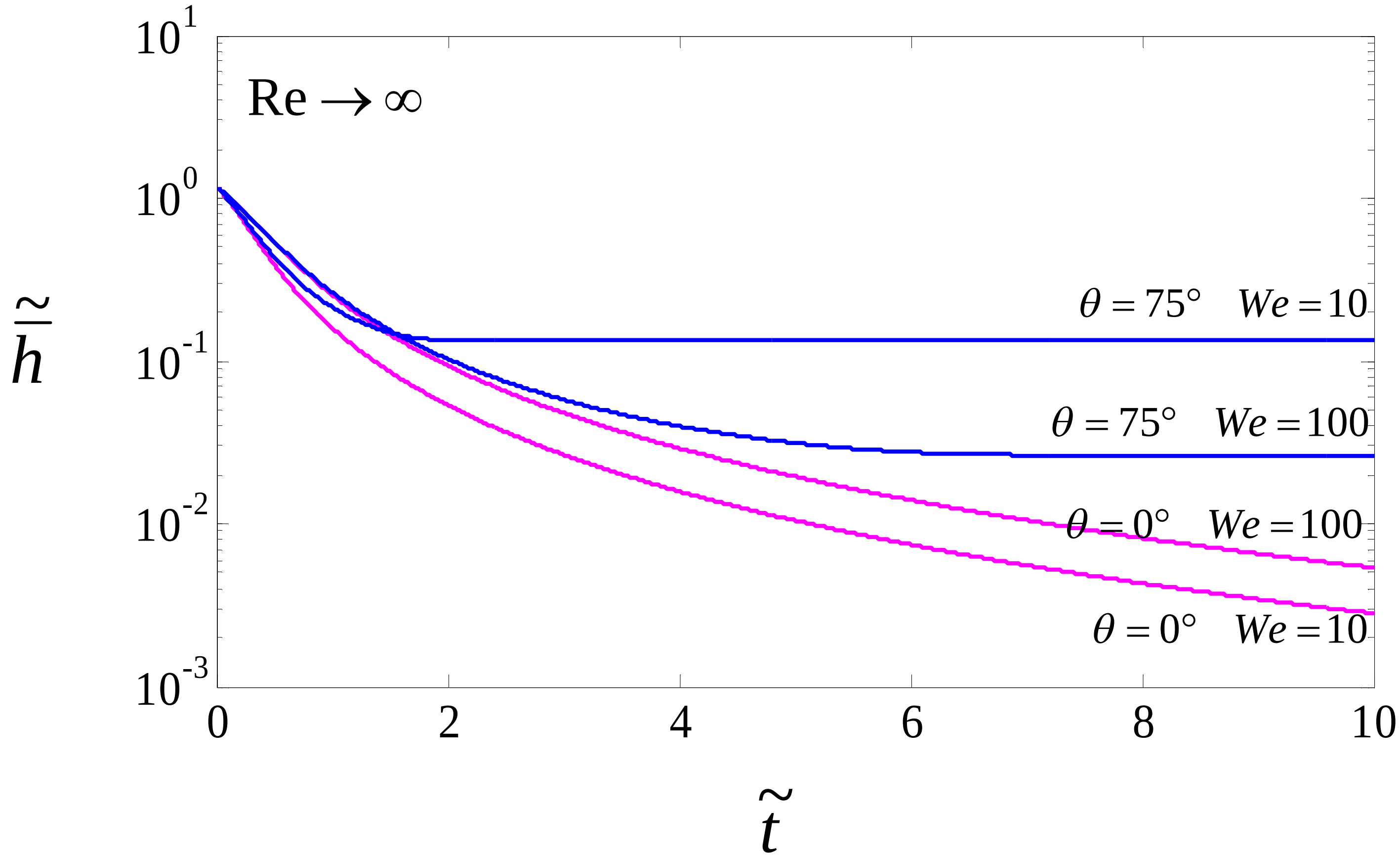
$$\dot{\tilde{h}} = -\tilde{h}\sqrt{6\frac{(We+12)\tilde{h} - 4\left(f_\theta + 2CWe\,\mathrm{Re}^{-1/2}\right)}{We\left(2\tilde{h}^3+1\right)}}$$

!!

$$f_\theta(T(t)) \equiv 1 + \frac{\sigma_{sl} - \sigma_{sg}}{\sigma_{\lg}} = 1 - \cos\theta \quad \mathrm{Re}(T(t)) \equiv \frac{\rho VD}{\mu} \quad We(T(t)) \equiv \frac{\rho V^2 D}{\sigma_{\lg}}$$

Droplet Impingement

<u>Splat thickness</u>



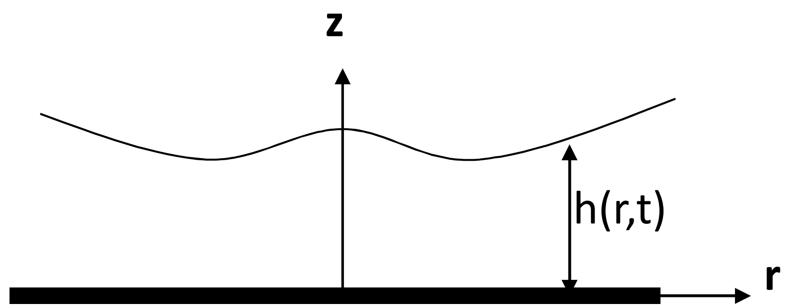
Droplet Impingement

Full N.S. based calculation

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)-\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{\partial^2 v_r}{\partial z^2}\right]=-\frac{\partial P}{\partial r}$$

$$=\sigma_{\lg}\frac{\partial}{\partial r}\left[\frac{h'}{r\sqrt{1+h'^2}}+\frac{h''}{\left(1+h'^2\right)^{3/2}}\right]$$

$$\frac{\partial(rh)}{\partial t}+\frac{\partial(rhv_r)}{\partial r}=0$$



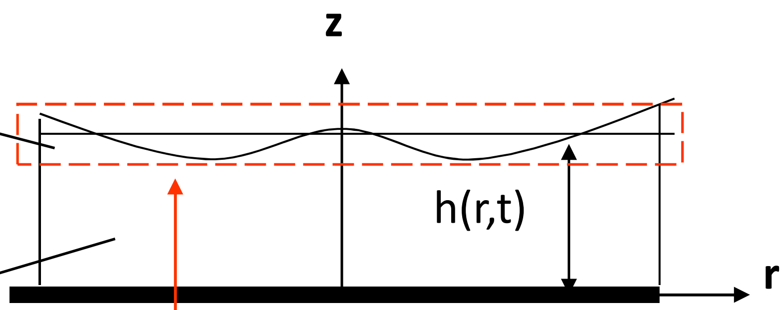
$$h(r,t)= f\left(r,t,D_{initial},v_{impact},properties\right)$$

Droplet Impingement

Suggested model

Small deformation: $h' << 1$

Free surface: $\frac{dv}{dz} \approx 0$



Bulk $\left(\bar{h}(t)\right)$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r}\right) = -\frac{\partial P}{\partial r} = \sigma_{\lg} \frac{\partial}{\partial r}\left(\frac{h'}{r} + h''\right)$$

Potential flow: $v_r \approx -\frac{1}{2}\frac{r}{\bar{h}}\dot{\bar{h}}$ $\quad v_z \approx \frac{z}{\bar{h}}\dot{\bar{h}}$

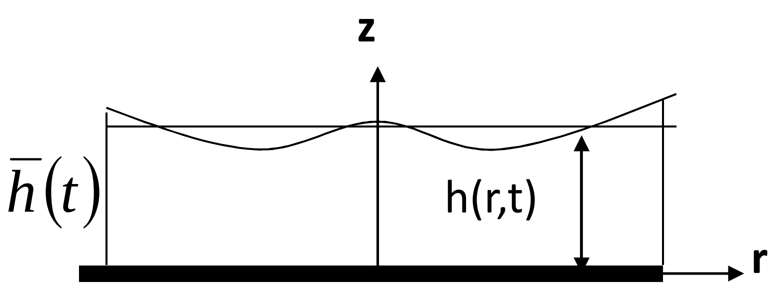
$$\Downarrow$$

$$h(r,t) = h\left(r, \bar{h}(t)\right) = f\left(r, t, D_{initial}, v_{impact}, properties\right)$$

Droplet Impingement

Suggested model – contd.

$$r^2 h''' + r h'' - h' = \frac{\rho}{\sigma_{lg}} \beta r^3 \qquad \beta \equiv \left(-\frac{1}{2}\frac{\ddot{\bar{h}}}{\bar{h}} + \frac{1}{4}\frac{\dot{\bar{h}}^2}{\bar{h}^2}\right)$$



$$\begin{cases} r = 0, \quad h = \bar{h} \\ r = 0, \quad h' = 0 \\ r = R, \quad p = p_0 + \sigma_{lg}\left(\dfrac{1}{R} + \dfrac{1-\cos\theta}{\bar{h}}\right) \Rightarrow \dfrac{h'}{r} + h'' = \dfrac{1}{R} + \dfrac{1-\cos\theta}{\bar{h}} \end{cases}$$

$$\dot{\tilde{\bar{h}}} = -\tilde{\bar{h}}\sqrt{6\frac{(We+12)\tilde{\bar{h}} - 4\left(f_\theta + 2CWe\,\mathrm{Re}^{-1/2}\right)}{We\left(2\tilde{\bar{h}}^3 + 1\right)}}$$

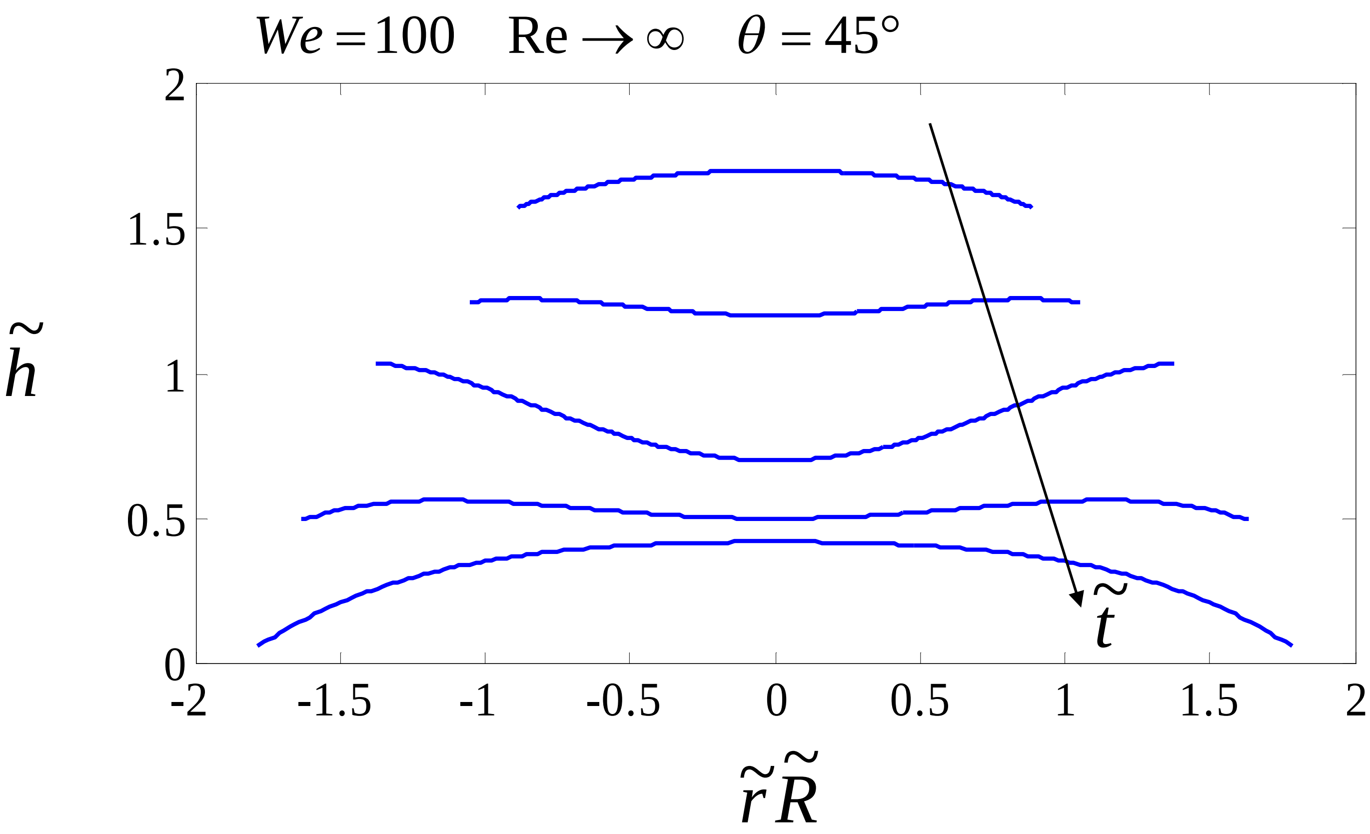
$$\tilde{R}^2 \tilde{\bar{h}} = \frac{4}{3}$$

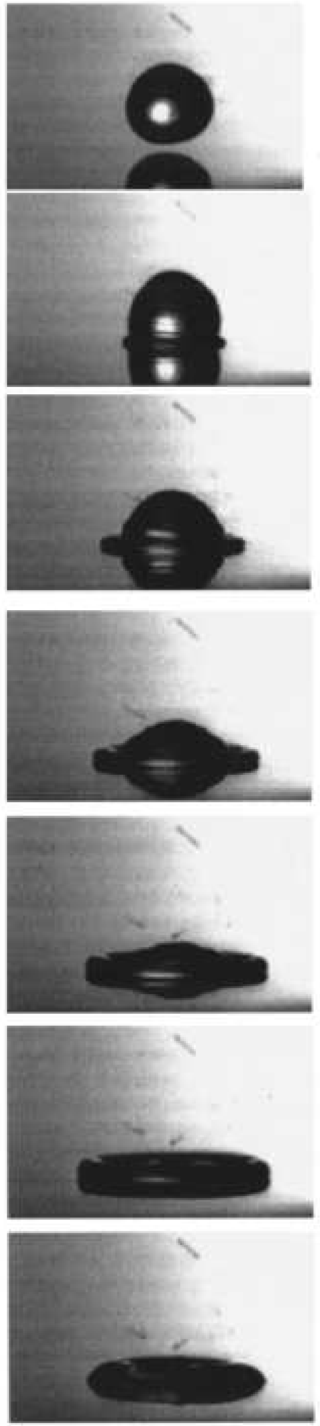
$$\tilde{h} = \frac{1}{144}\frac{\tilde{\beta}}{\tilde{\bar{h}}^2} We\left(\tilde{r}^4 - 4\tilde{r}^2\right) - \left(\frac{1}{2\sqrt{3}}\frac{1}{\sqrt{\tilde{\bar{h}}}} + \frac{1}{3}\frac{1-\cos\theta}{\tilde{\bar{h}}^2}\right)\tilde{r}^2 + \tilde{\bar{h}}$$

$$\tilde{\beta} = \frac{3}{2}\frac{1}{\left(2\tilde{\bar{h}}^3 + 1\right)^2}\frac{1}{We}\left[\left(f_\theta + 2CWe\,\mathrm{Re}^{-1/2}\right)\left(8\tilde{\bar{h}}^3 + 1\right) - \frac{3}{2}(We+12)\tilde{\bar{h}}^4\right]$$

Droplet Impingement

Calculated impingement

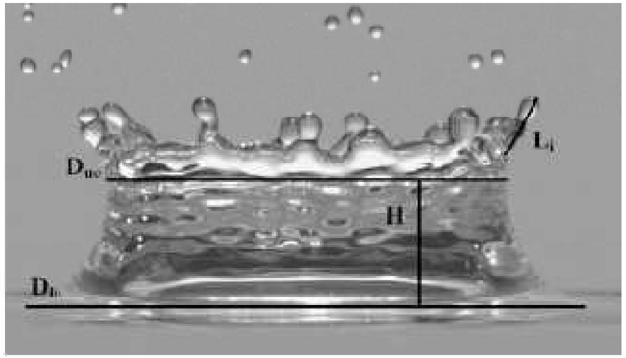


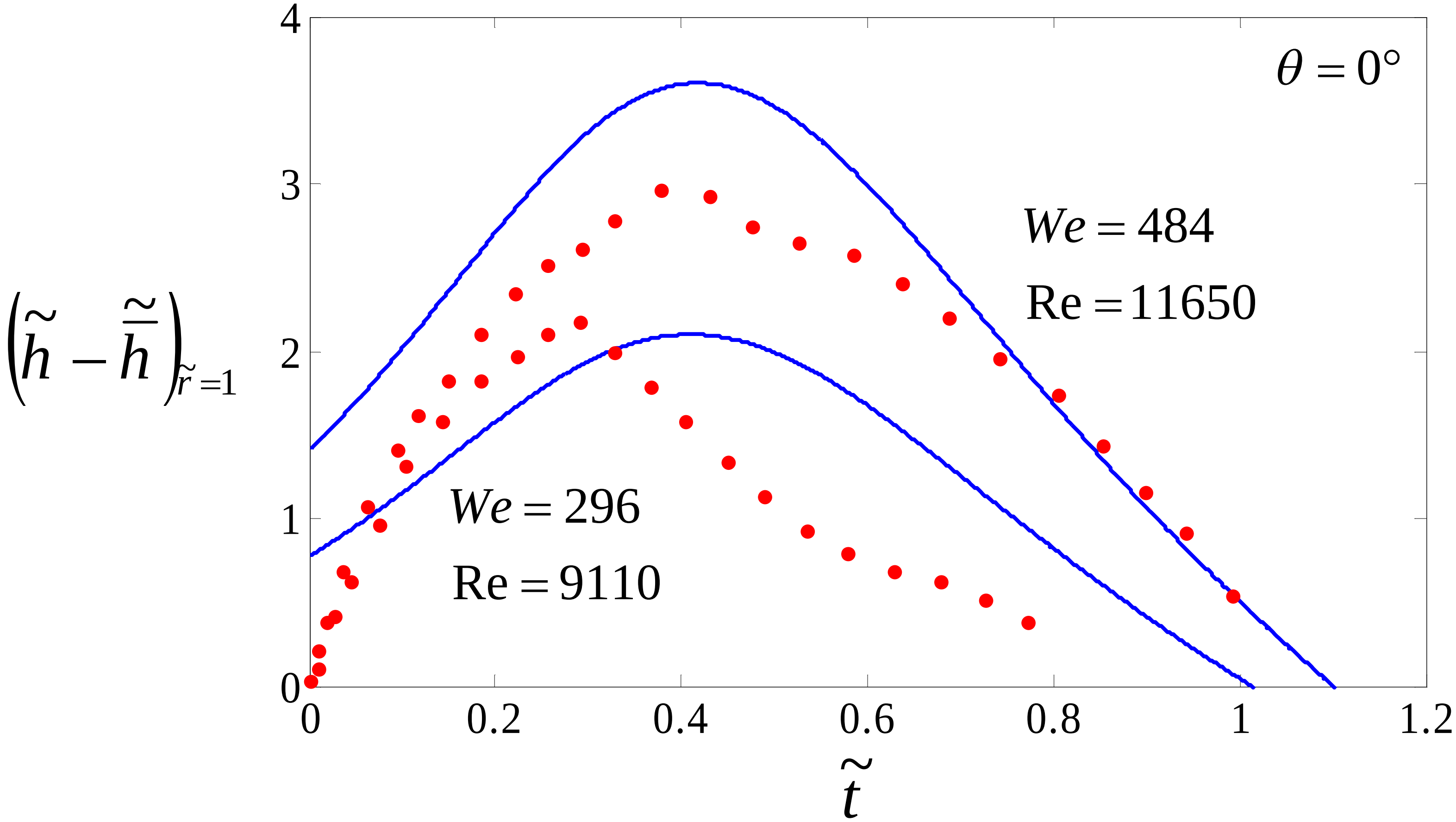


* Images from H.Y. Kim and J.H. Chun (2001)

Droplet Impingement

<u>Crowning</u>

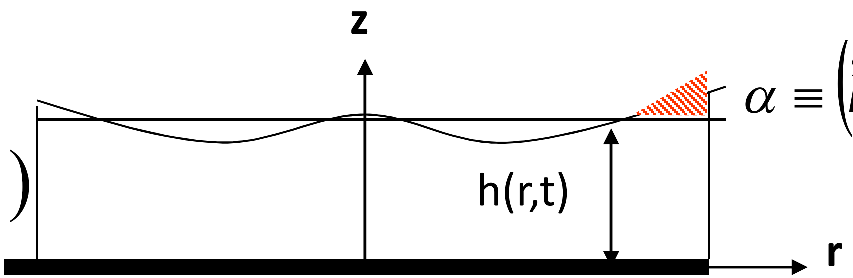




* Experimental data from Cossali et al (2004)

Droplet Impingement

Splashing



$$\alpha \equiv \left(\tilde{h} - \tilde{\tilde{h}}\right)_{\max}$$

$$\overline{h}\left(t\right)$$

$$\left.\frac{d\left(\tilde{h} - \tilde{\tilde{h}}\right)}{d\tilde{\tilde{h}}}\right|_{\tilde{r}=1} = 0 \quad \Longrightarrow \quad \left.\tilde{\tilde{h}}\right|_{\left(\tilde{h}-\tilde{\tilde{h}}\right)_{\max}} \approx 0.7$$

$$\alpha \equiv \left(\tilde{h} - \tilde{\tilde{h}}\right)_{\max} \approx \frac{1}{125}\left[We\left(1 - 20C\,\mathrm{Re}^{-1/2}\right) - 95\left(1 - \cos\theta\right) - 31\right]$$

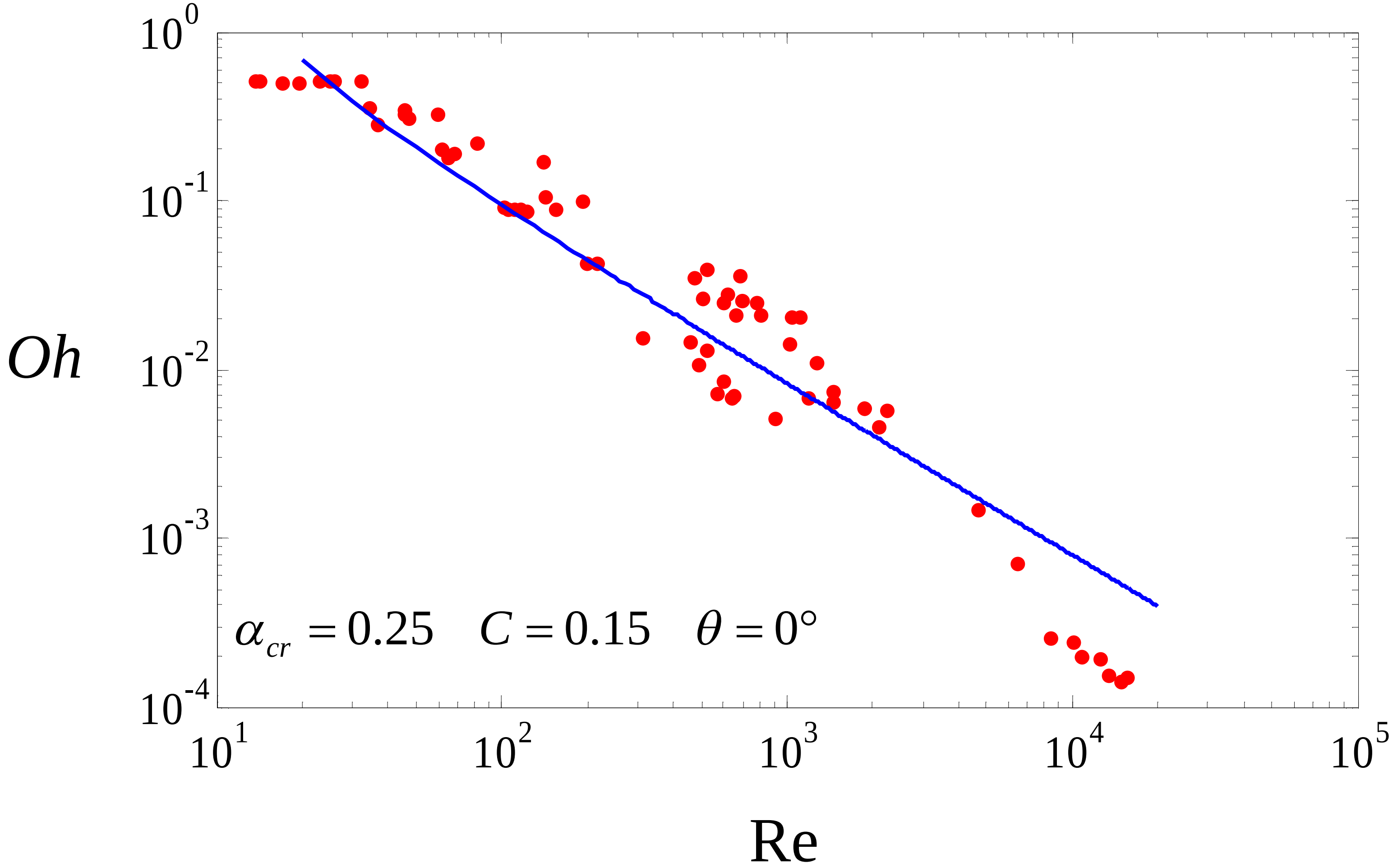
$$We > \frac{125\alpha_{cr} + 31 + 95\left(1 - \cos\theta\right)}{\left(1 - 20C\,\mathrm{Re}^{-1/2}\right)}$$

$$Oh \equiv \frac{\mu}{\sqrt{\rho\sigma D}} = \frac{\sqrt{We}}{\mathrm{Re}}$$

$$Oh > \frac{1}{\mathrm{Re}}\sqrt{\frac{125\alpha_{cr} + 31 + 95\left(1 - \cos\theta\right)}{\left(1 - 20C\,\mathrm{Re}^{-1/2}\right)}}$$

Droplet Impingement

Splashing



* Experimental data from Mundo et al (1995)

Droplet Impingement

Parametric effects on splashing

Surface

Wet → Dry hydrophobic

$$\theta = 0° \rightarrow 180°$$

Smooth → Rough

$$\cos\theta_{eq} \rightarrow r_w \cos\theta$$

(Wenzel)

$$T_s = 20°C \rightarrow 300°C$$

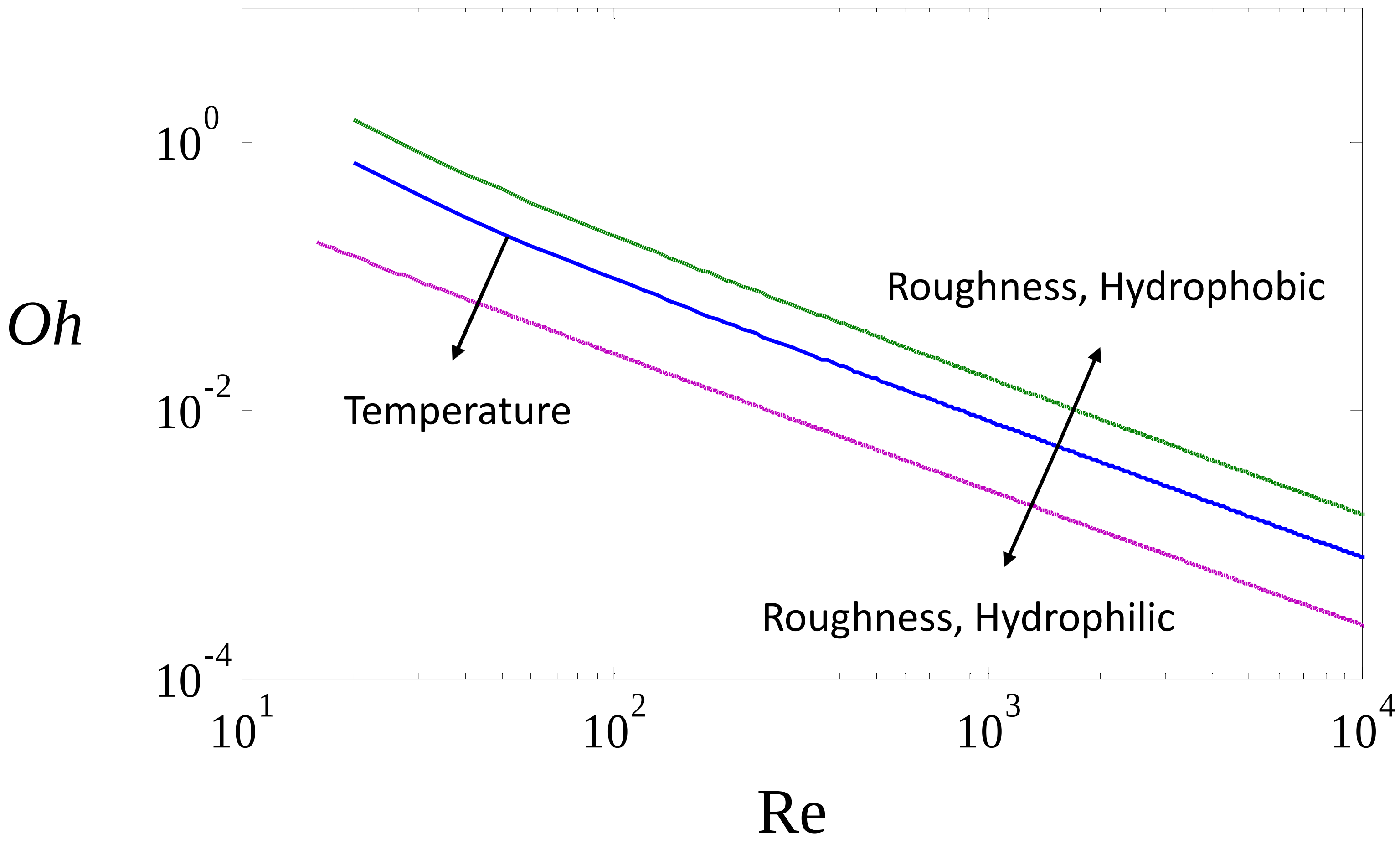
$$\Downarrow$$

$$\sigma = 70 mN/m \rightarrow 7 mN/m$$

$$\mu = 1000 \mu Pa \cdot s \rightarrow 80 \mu Pa \cdot s$$

Droplet Impingement

Surface effect on splashing



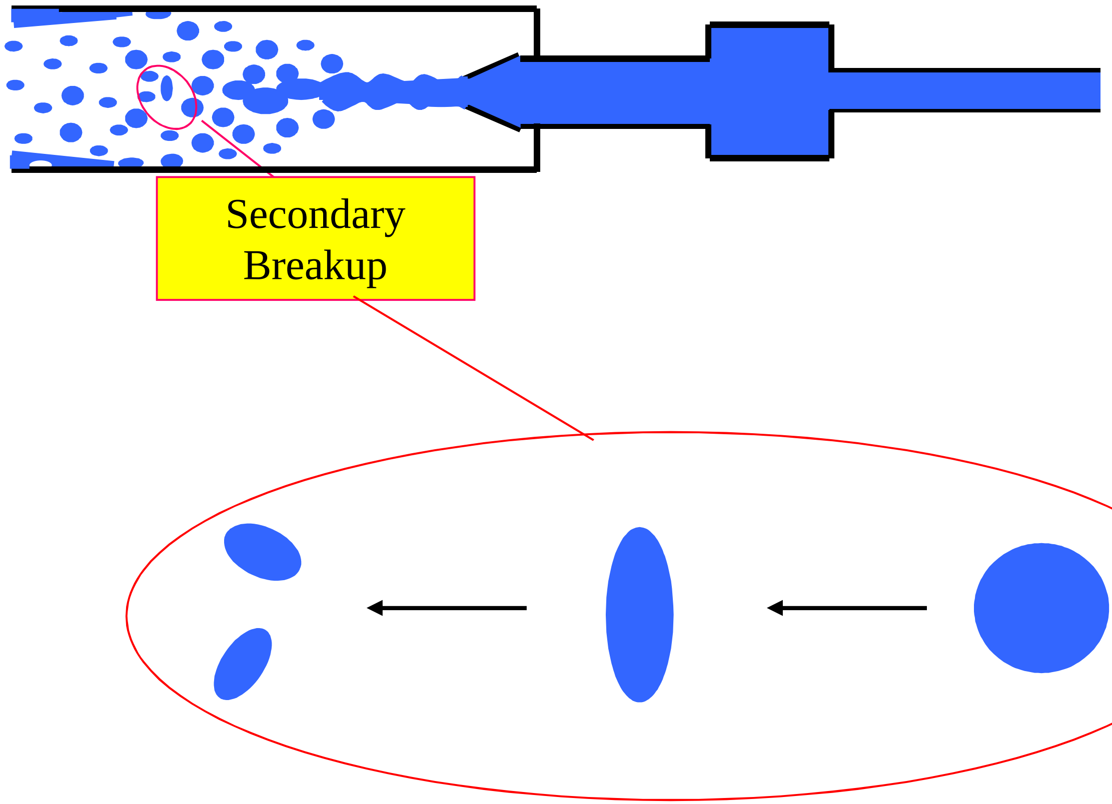
Droplet Impingement

Simple analytical model:

- Qualitative deformation prediction
- Quantitative “crowning” prediction
- Quantitative splash prediction
- Analytical transient temperature

Droplet Breakup

Transient trajectory velocity



Droplet Breakup

Energy balance

$$E_s = n \cdot e_s + E_{diss.}$$

Initial surface energy (Meiron, 1989):

$$R(\theta) = r_e\left[1 - \frac{3}{64}We(3\cos\theta + 1)\right]$$

$$We \equiv \frac{\rho_g U^2 r_e}{\sigma}$$

$$r_e \equiv \left(\frac{3V}{4\pi}\right)^{1/3}$$

$$E_s = \sigma \cdot 2\pi \int_0^{\pi} R^2(\theta)\sin\theta d\theta = \sigma\pi r_e^2\left(\frac{3^2}{5\cdot 2^6}We^2 + 4\right)$$

Final surface energy:

$$e_s = \sigma \cdot n^{-2/3} \cdot 4\pi r_e^2$$

Dissipated energy:

$$E_{diss.} = \mu\left(\frac{du}{dx}\right)^2 \cdot V = \mu\left(\frac{r_e/t}{r_e}\right)^2 \cdot \frac{4}{3}\pi r_e^3 \cdot t = \frac{4}{3}\pi r_e^3 \mu \frac{1}{t}$$

Droplet Breakup

Droplet velocity

Droplet's dynamic equation:

$$\frac{dU}{dt} = -\frac{3}{8} C_D \frac{\rho_g U^2}{\rho_l r_e}$$

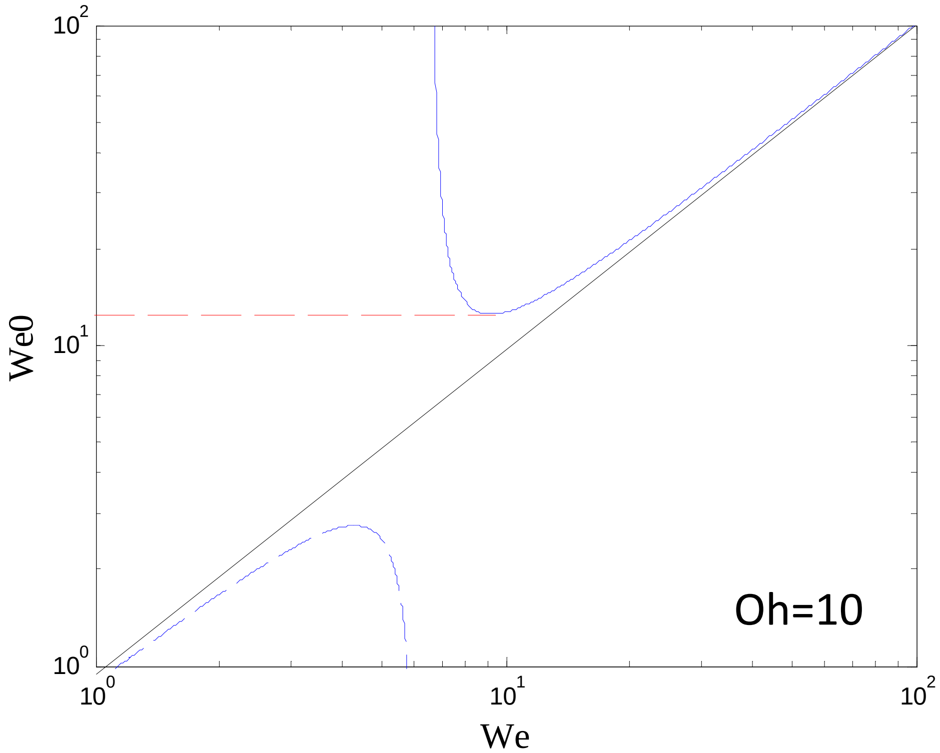
$$\frac{1}{t} = \frac{\frac{3}{8} C_D \frac{\rho_g U_0}{\rho_l r_e}}{\frac{\sqrt{We_0}}{\sqrt{We}} - 1} = \frac{3}{8} \frac{\sigma}{\mu_l r_e} C_D \sqrt{\frac{\rho_g}{\rho_l}} Oh \frac{\sqrt{We_0}\sqrt{We}}{\sqrt{We_0} - \sqrt{We}}$$

$$Oh \equiv \frac{\mu_l}{\sqrt{\rho_l r_e \sigma}}$$

Droplet Breakup

Breakup conditions

Relation between initial and at breakup conditions: $\frac{3^2}{5\cdot 2^8}We^2=\left(n^{1/3}-1\right)+\frac{1}{8}C_D\sqrt{\frac{\rho_g}{\rho_l}}Oh\frac{\sqrt{We_0}\sqrt{We}}{\sqrt{We_0}-\sqrt{We}}$



Critical breakup at: $\left.\frac{d\left(\sqrt{We_0}\right)}{d\left(\sqrt{We}\right)}\right|_{We=We_{cr}}=0$

Droplet Breakup

Critical breakup conditions

Asymptotic solutions:

$$\left(n^{1/3}-1\right) << \frac{3^2}{5\cdot 2^8} We_{cr}^2$$

$$C_D\sqrt{\frac{\rho_g}{\rho_l}}Oh \to 0$$

$$We_{cr} = \left(\frac{5\cdot 2^7}{3^2}C_D\sqrt{\frac{\rho_g}{\rho_l}}Oh\right)^{2/3} = \left(\frac{3}{4}\right)^2 We_{0,cr}$$

$$We_{cr} \to We_{0,cr}$$

$$We_{0,cr} = \left(\frac{5\cdot 2^{13}}{3^5}C_D\sqrt{\frac{\rho_g}{\rho_l}}Oh\right)^{2/3}$$

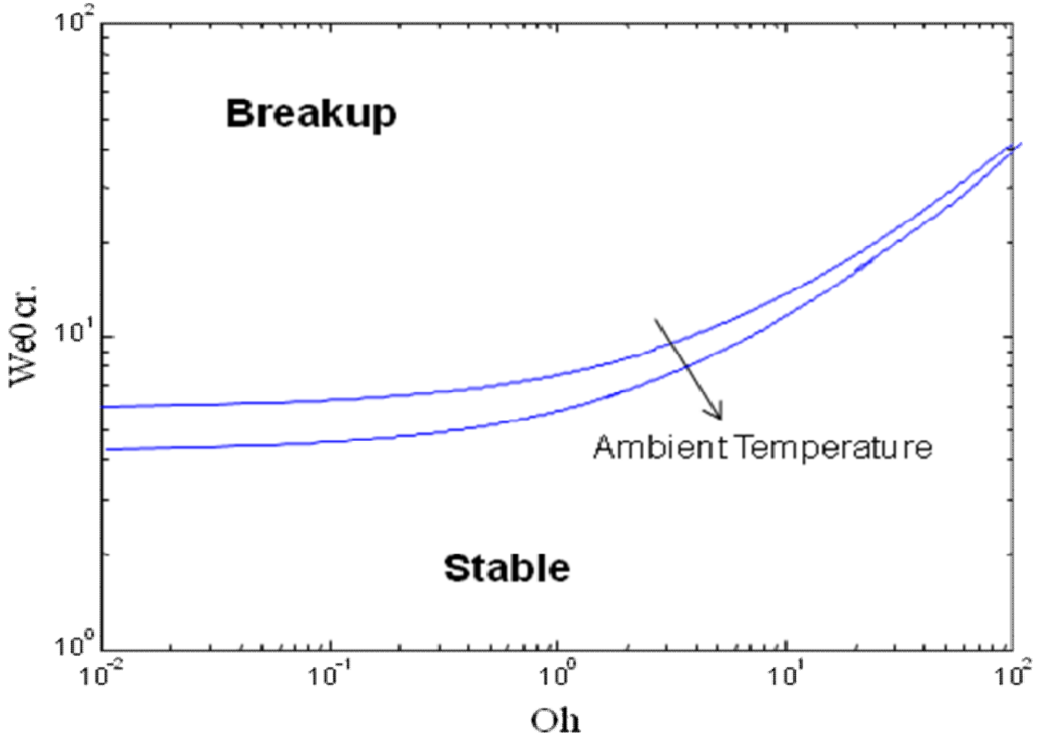
$$We_{0,cr} \to \frac{\sqrt{5}\cdot 2^4}{3}\sqrt{\left(n^{1/3}-1\right)}$$

Combined solution:

$$We_{0,cr} = 11.93\sqrt{\left(n^{1/3}-1\right)} + 30.51\left(C_D\sqrt{\frac{\rho_g}{\rho_l}}Oh\right)^{2/3}$$

Droplet Breakup

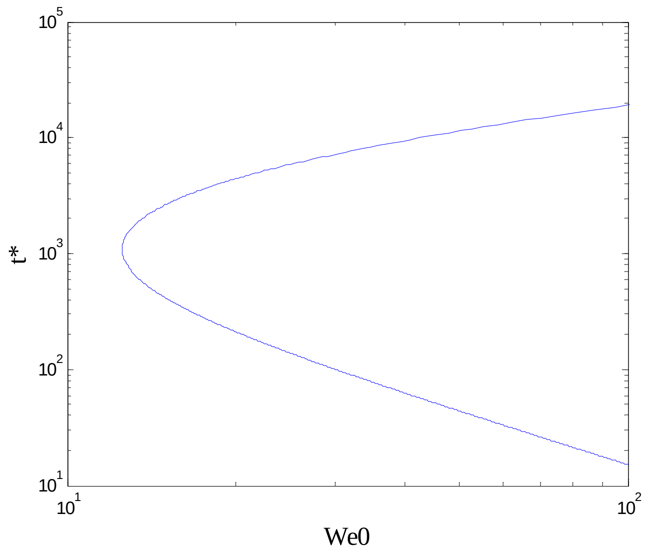
Critical breakup results



Droplet Breakup

Breakup time

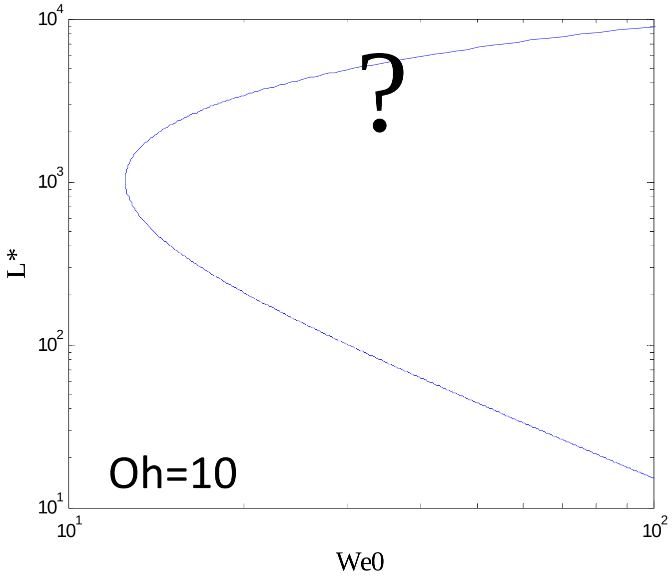
$$t^* \equiv \frac{U_0 t}{r_e}$$



Oh=10

Travel distance until breakup

$$L^* \equiv \frac{L}{r_e} = \frac{1}{r_e}\int_0^t U dt = \frac{1}{\frac{3}{8}C_D\frac{\rho_g}{\rho_l}}\ln\left(\frac{3}{8}C_D\frac{\rho_g}{\rho_l}t^* + 1\right)$$



Droplet Breakup

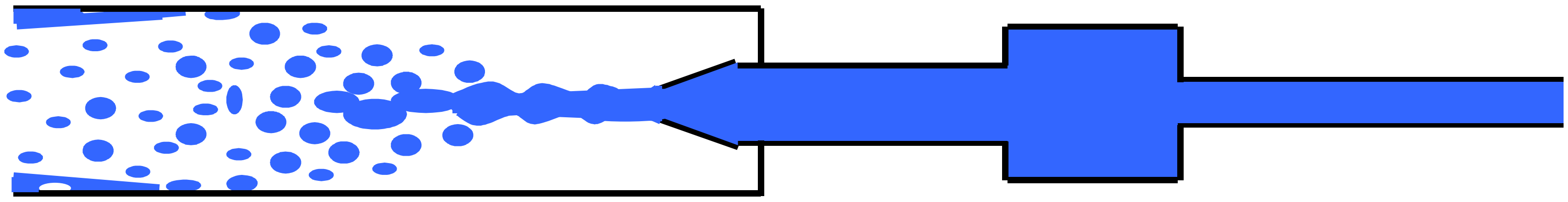
-A simple analytic approach to analyse breakup limits of a droplet with transient trajectory velocity.

-Spray droplet size distribution ?

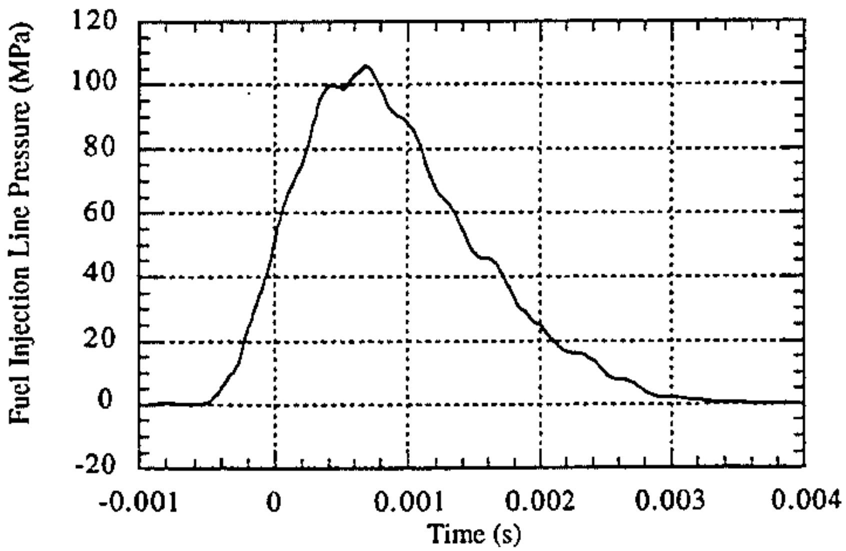
- Transient surface tension (injection to hot ambient)?

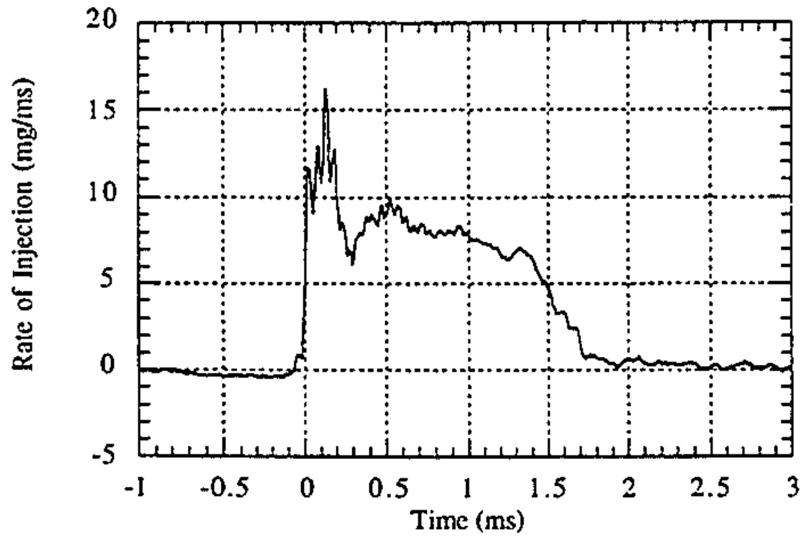
Jet Breakup

Transient velocity



Fuel injection in a Diesel engine is a transient process





* C.C. Hung and J.K. Martin, J.Y. Koo, SAE-970053, 1997

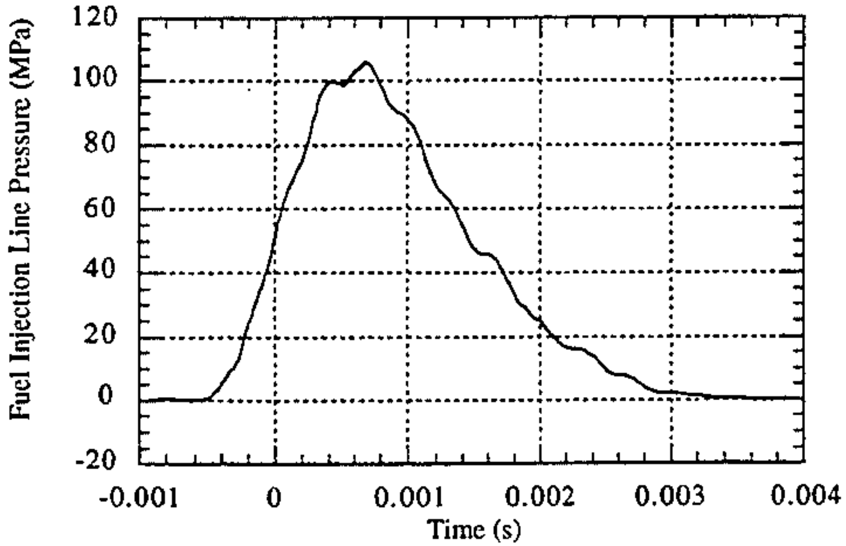
Jet Breakup

Inertial stresses

A jet changing velocity 0-100m/s in 1ms:

$$dp/dx \approx \rho_l \cdot a \approx 800\,kg/m^3 \cdot \frac{100\,m/s}{1ms} = 80MPa$$

$$dp/dx \approx \sigma/d^2 \approx \frac{13\,mN/m}{(10\,\mu m)^2} = 130MPa$$



Jet Breakup

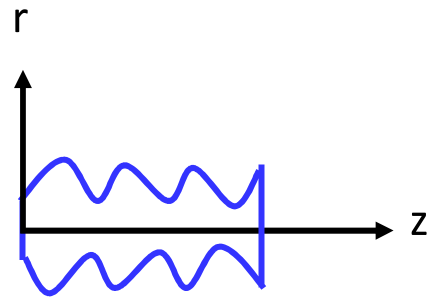
Basic equations

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta)+\frac{\partial}{\partial z}(u_z)=0$$

z - Momentum:

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\nabla^2 u_z$$



r - Momentum:

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+u_z\frac{\partial u_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\nabla^2 u_r$$

Instability perturbations:

$$u_z=\overline{u}_z+u_z' \qquad u_r=\overline{u}_r+u_r' \qquad p=\overline{p}+p' \qquad R=R_0+\delta$$

Jet Breakup

Perturbations form

For a transient inviscid jet, where: $\bar{u}_z(t)$

Continuity: $$\frac{1}{r}\frac{\partial}{\partial r}(ru'_r)+\frac{\partial}{\partial z}(u'_z)=0$$

$$\frac{\partial \bar{u}_z}{\partial z}=\frac{1}{\bar{u}_z}\frac{\partial \bar{u}_z}{\partial t}\rightarrow 0$$

z-Momentum: $$\rho\left(\frac{\partial u'_z}{\partial t}+\bar{u}_z\frac{\partial u'_z}{\partial z}+u'_z\frac{\partial \bar{u}_z}{\partial z}\right)=-\frac{\partial p'}{\partial z}$$

r-Momentum: $$\rho\frac{\partial u'_r}{\partial t}=-\frac{\partial p'}{\partial r}$$

This analysis:

$$\delta = Ae^{\beta(t)+i\alpha z}$$

$$p' = f(r)\hat{p}(t)e^{\beta(t)+i\alpha z}$$

$$u'_z = f(r)\hat{u}(t)e^{\beta(t)+i\alpha z}$$

Classical analysis:

$$\delta = Ae^{\beta\cdot t+i\alpha z}$$

$$p' = f(r)\hat{p}e^{\beta\cdot t+i\alpha z}$$

$$u'_z = f(r)\hat{u}e^{\beta\cdot t+i\alpha z}$$

Jet Breakup

Kinematic Boundary Condition

$$u'_r\Big|_{r=R_0} = \frac{\partial\delta}{\partial t} + \bar{u}_z \frac{\partial\delta}{\partial z}$$

For vapour region (outer): $$\hat{u} = -\frac{A}{K_1(\alpha R_0)}\left[i\frac{d\beta(t)}{dt} - \alpha\bar{u}_z\right]$$

For liquid region (inner): $$\hat{u} = -\frac{A}{I_1(\alpha R_0)}\left[i\frac{d\beta(t)}{dt} - \alpha\bar{u}_z\right]$$

Jet Breakup

Dynamic Boundary Condition

$$\left(p_l' - p_v'\right)\Big|_{r=R_0} = \sigma\left(-\frac{\partial^2\delta}{\partial x^2} + \frac{1-\delta/R_0}{R_0}\right)$$

Substituting perturbation parameters:

$$f\left(R_0\right)\left[\hat{p}_l\left(t\right) - \hat{p}_v\left(t\right)\right] = \sigma A\left(\alpha^2 - \frac{1}{R_0^{\ 2}}\right)$$

Jet Breakup

The dispersion relation

Subtracting the momentum equations of the vapour from the liquid phase, at the surface (for a moving liquid in ambient gas at rest):

$$\left(\frac{I_0(\alpha R_0)}{I_1(\alpha R_0)}\rho_l + \frac{K_0(\alpha R_0)}{K_1(\alpha R_0)}\rho_v\right)\left[\frac{d^2\beta(t)}{dt^2} + \left(\frac{d\beta(t)}{dt}\right)^2\right] + \frac{I_0(\alpha R_0)}{I_1(\alpha R_0)}\rho_l\alpha\left(2i\bar{u}_z\frac{d\beta(t)}{dt} + i\frac{d\bar{u}_z}{dt} - \alpha\bar{u}_z^{\,2}\right) + \sigma\alpha\left(\alpha^2 - \frac{1}{R_0^{\,2}}\right) = 0$$

For expected short waves: $\alpha R_0 \to \infty \quad \Rightarrow \quad \frac{I_1(\alpha R_0)}{I_0(\alpha R_0)} \approx 1, \frac{K_1(\alpha R_0)}{K_0(\alpha R_0)} \approx 1$

$$(\rho_l + \rho_v)\left[\frac{d^2\beta(t)}{dt^2} + \left(\frac{d\beta(t)}{dt}\right)^2\right] + \rho_l\alpha\left(2i\bar{u}_z\frac{d\beta(t)}{dt} + i\frac{d\bar{u}_z}{dt} - \alpha\bar{u}_z^{\,2}\right) + \sigma\alpha\left(\alpha^2 - \frac{1}{R_0^{\,2}}\right) = 0$$

Jet Breakup

Simplified solution – steady jet

For a steady jet: $$\frac{d\bar{u}_z}{dt} = 0, \quad \frac{d\beta(t)}{dt} \rightarrow const.$$

$$\frac{d\beta(t)}{dt} = \frac{-i\alpha\rho_l\bar{u}_z \pm \sqrt{\alpha\rho_l\rho_v\bar{u}_z^{\,2} - (\rho_l + \rho_v)\alpha\sigma\left(\alpha^2 - \frac{1}{R_0^{\,2}}\right)}}{\rho_l + \rho_v}$$

The dominant wave is expected to be at: $$\frac{\partial}{\partial\alpha}\mathrm{Re}\left(\frac{d\beta(t)}{dt}\right) = 0$$

$$\alpha = \frac{\bar{u}_z^{\,2} + \sqrt{\bar{u}_z^{\,4} - \left(\frac{\rho_l + \rho_v}{\rho_l\rho_v}\sigma\right)^2 \frac{3}{R_0^{\,2}}}}{3\frac{\rho_l + \rho_v}{\rho_l\rho_v}\sigma}$$

$$\alpha >> \frac{1}{R_0} \longrightarrow \alpha = \frac{2}{3}\frac{\rho_l\rho_v}{\rho_l + \rho_v}\frac{\bar{u}_z^{\,2}}{\sigma}$$

$$\bar{u}_z = 0 \longrightarrow \alpha = \frac{1}{\sqrt{3}R_0}$$

Jet Breakup

Simplified solution – transient jet

For transient jets of interest, wave growth is more rapid than velocity change, so for the time until breakup:

$$\bar{u}_z \approx const., \quad \frac{d\bar{u}_z}{dt} = a, \quad \frac{d\beta(t)}{dt} \to const.$$

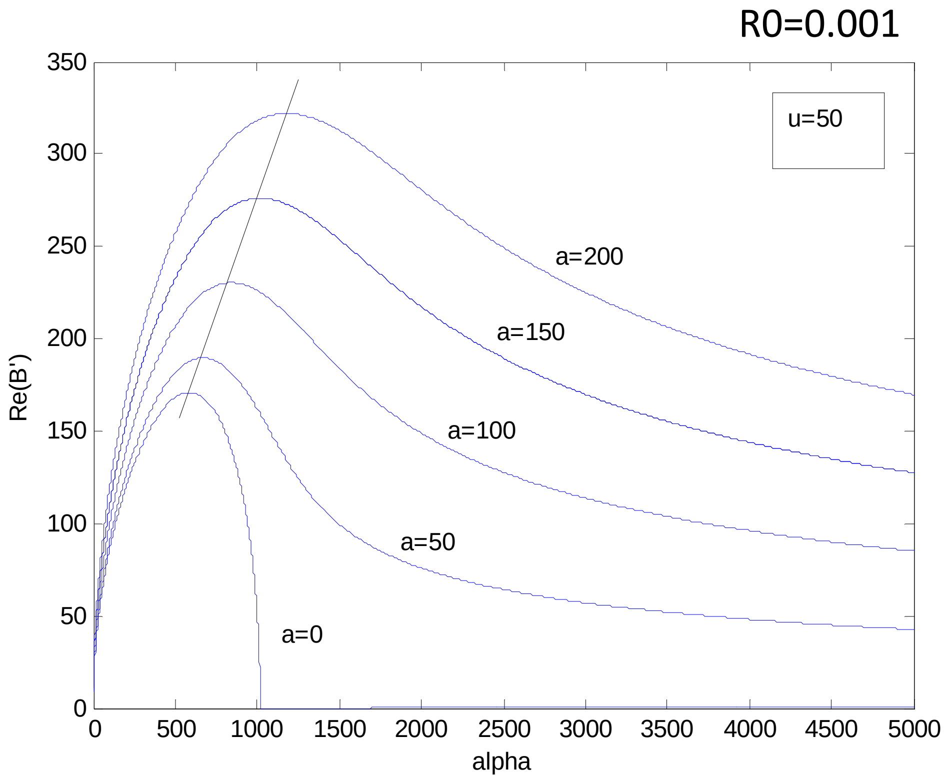
$$\frac{d\beta(t)}{dt} = \frac{-i\alpha\rho_l\bar{u}_z \pm \sqrt{\alpha\rho_l\rho_v\bar{u}_z^{\,2} - (\rho_l+\rho_v)\alpha\sigma\left(\alpha^2 - \frac{1}{R_0^{\,2}}\right) - i(\rho_l+\rho_v)\rho_l\alpha a}}{\rho_l+\rho_v}$$

$$\mathrm{Re}\left(\frac{d\beta(t)}{dt}\right) = \frac{\sqrt{\alpha}\sqrt{\sqrt{\left[\rho_l\rho_v\bar{u}_z^{\,2} - (\rho_l+\rho_v)\sigma\left(\alpha^2 - \frac{1}{R_0^{\,2}}\right)\right]^2 + [(\rho_l+\rho_v)\rho_l a]^2} + (\rho_l+\rho_v)\rho_l a}}{\sqrt{2}(\rho_l+\rho_v)}$$

Jet Breakup

Transient jet – Results1

Effect of acceleration

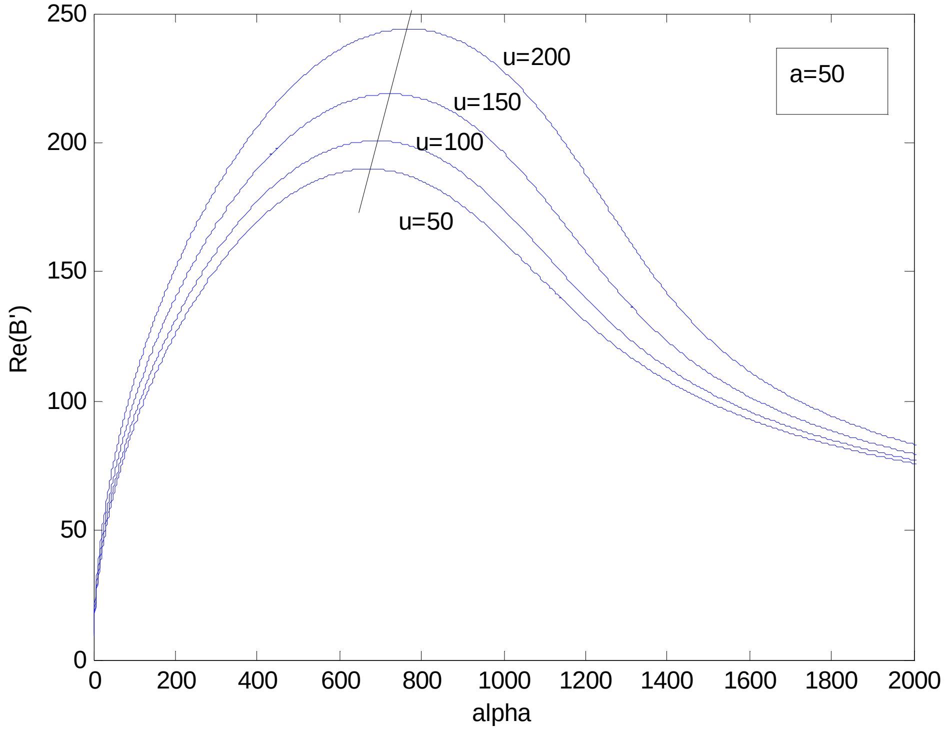


Jet Breakup

Transient jet – Results2

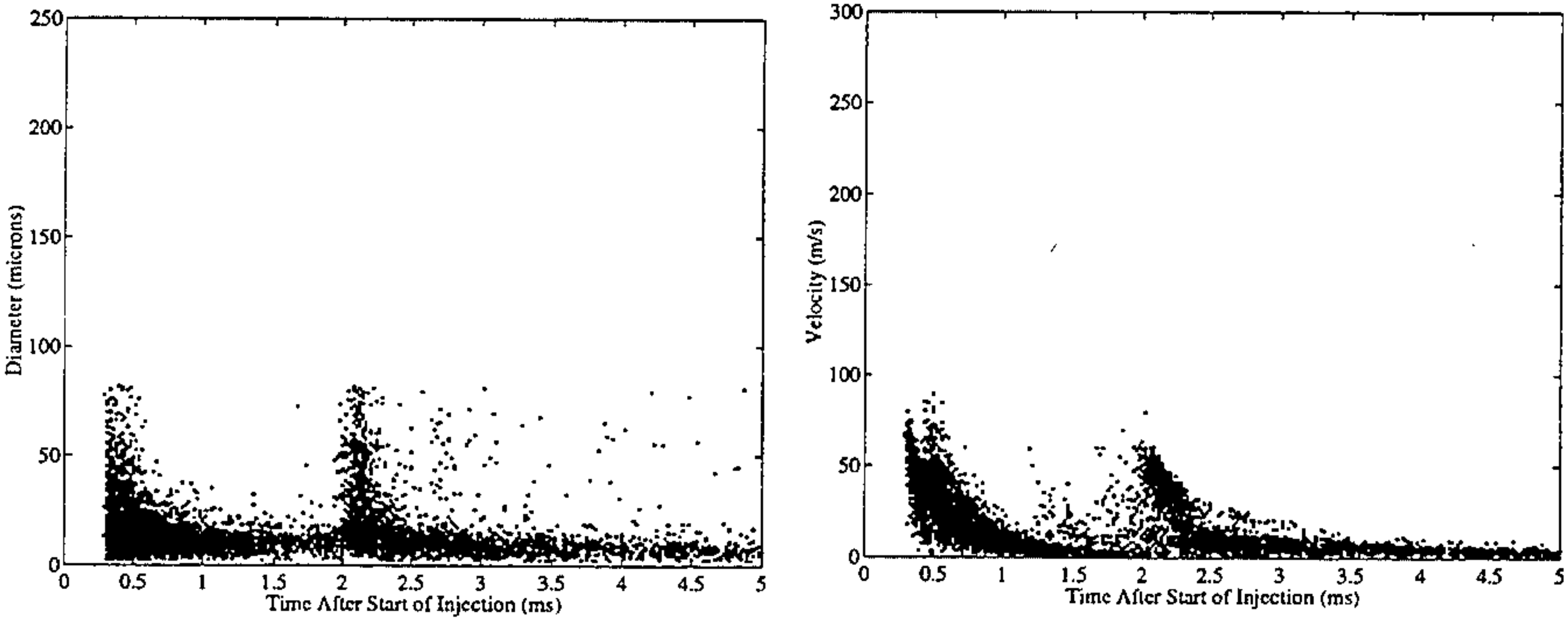
Effect of velocity

R0=0.001



Verification

Available experimental results*



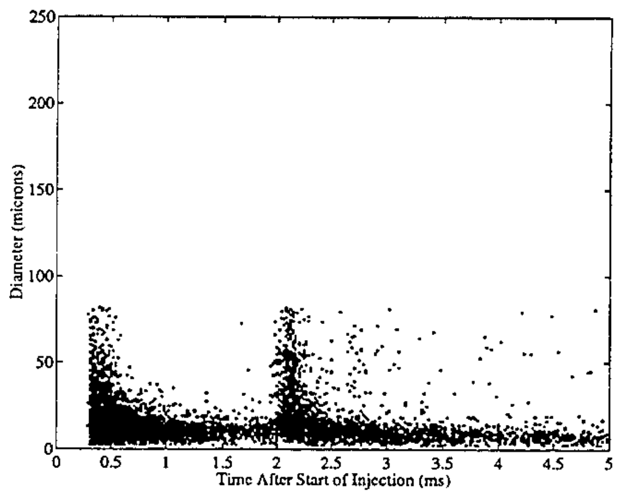
Droplet velocity and diameter profiles for the 21 MPa injection pressure spray at the position $z = 3$ cm, $r = 3$ mm.

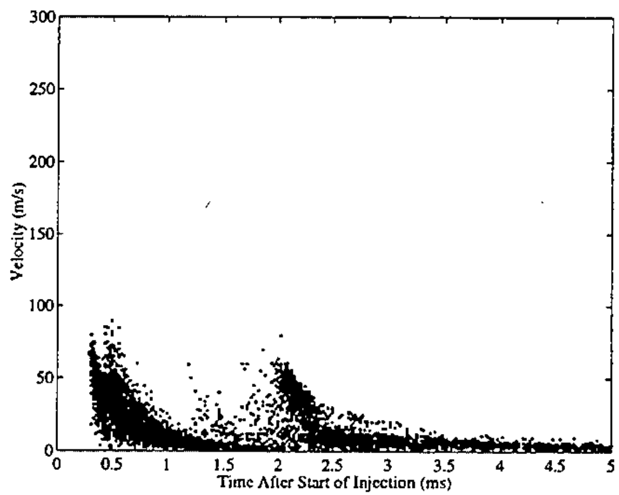
* C.C. Hung and J.K. Martin, J.Y. Koo, SAE-970053, 1997

Verification

Featured aspects in acceleration/deceleration:

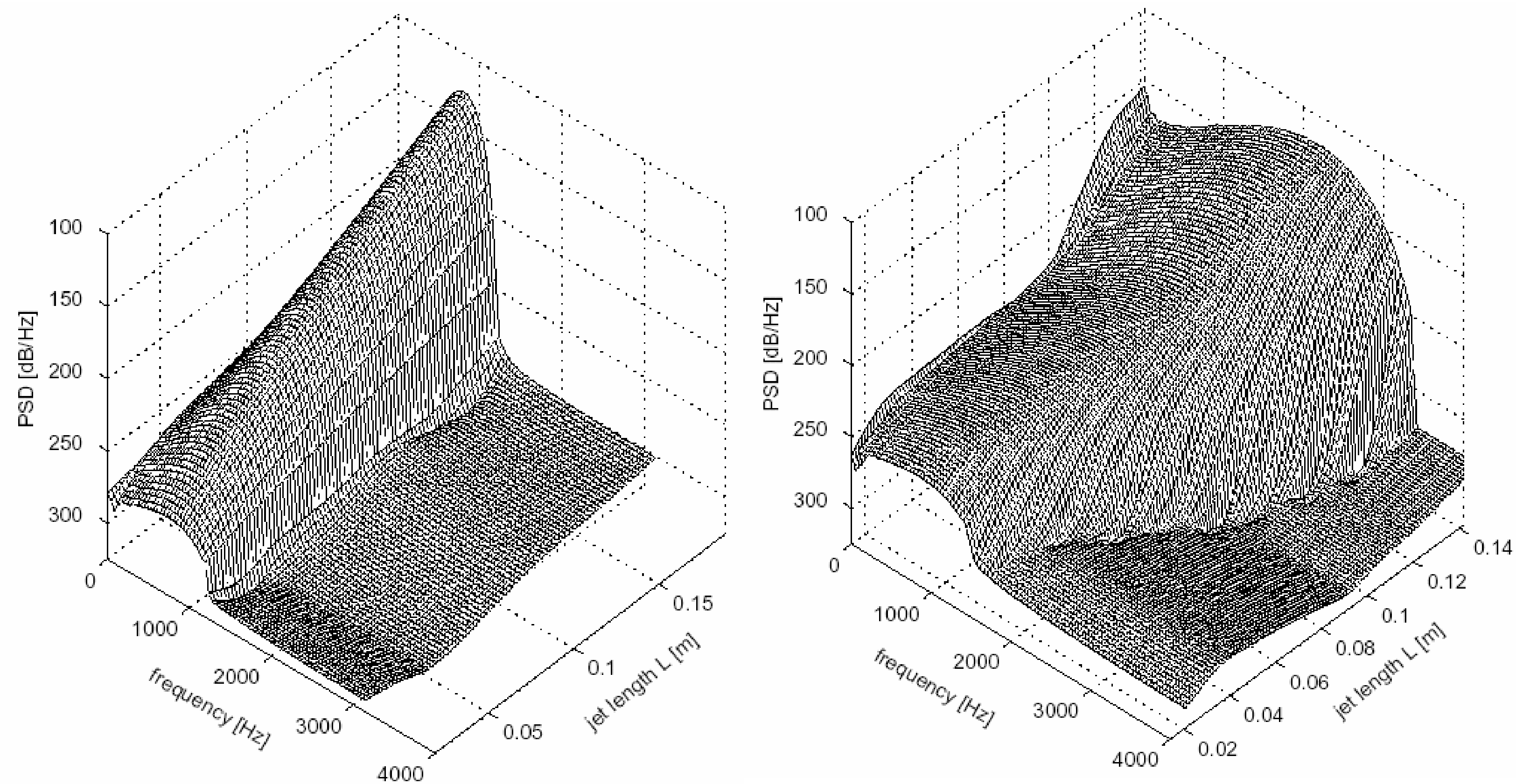
- Increased span of diameters in
- Increased growth rate (~drops velocity)
- Invariance to whether accelerating or decelerating





Verification-2

- As there is a lack in available experimental results on that subject, comparison is also sought with available numerical results*

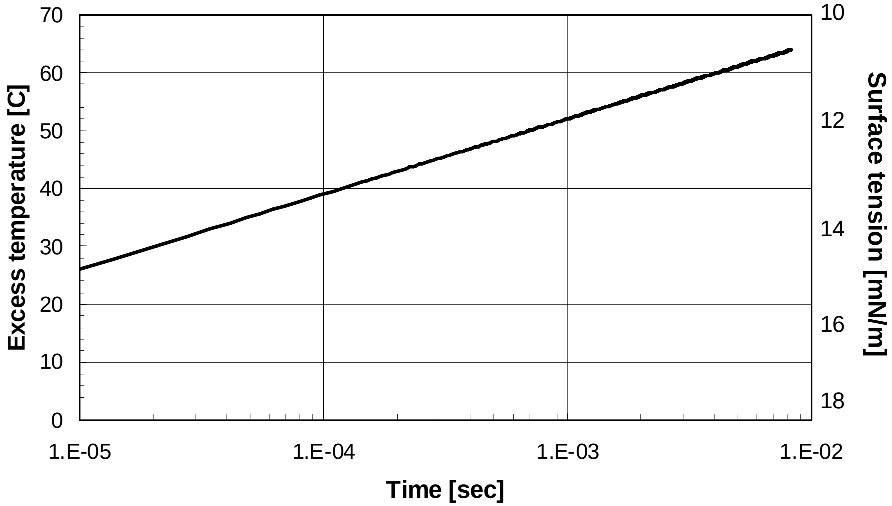


* R. Domann and Y. Hardalupas, AIAA-2004-1105, 42nd AIAA conf., 2004

Further application – Non-isothermal jets

Jet velocity ~ 10 - 100m/s
Jet diameter ~ 1mm
Ambient temperature excess ~ 1000K

Jet surface excess temperature

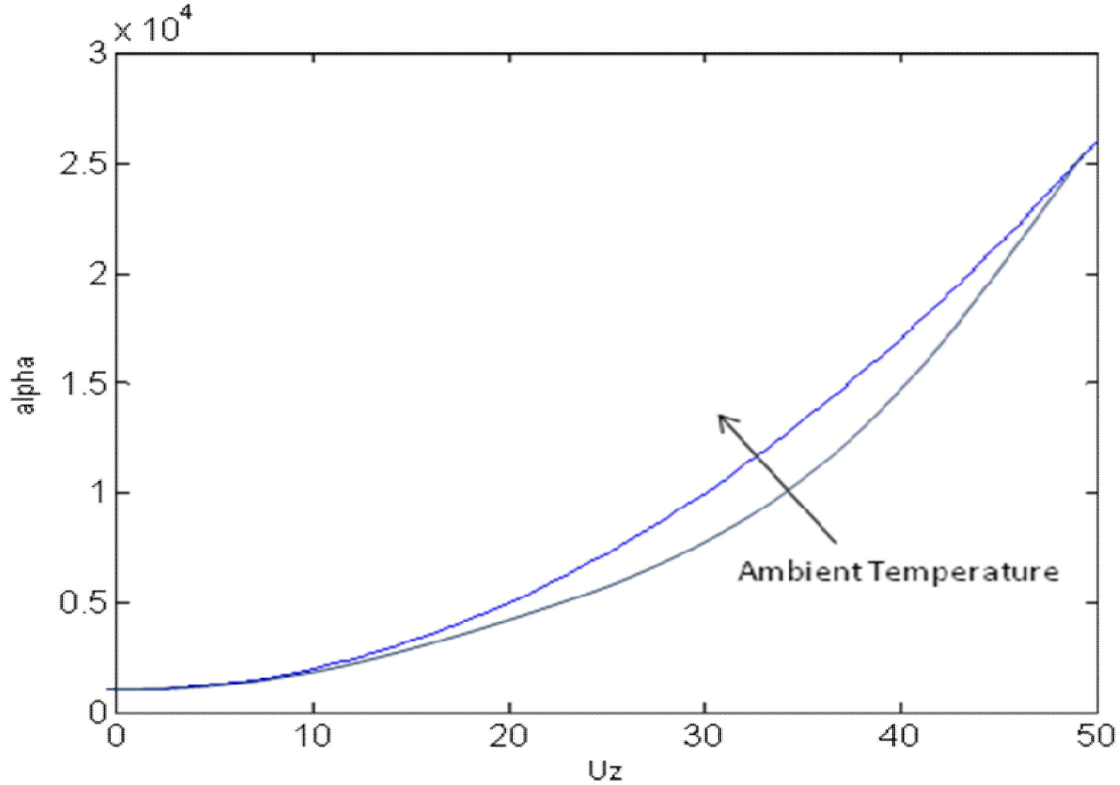


Coordinate system moves with the liquid phase: $\sigma(T) \rightarrow \sigma(t)$

$$(\rho_l + \rho_v)\left[\frac{d^2\beta(t)}{dt^2} + \left(\frac{d\beta(t)}{dt}\right)^2\right] + \rho_l\alpha\left(2i\bar{u}_z\frac{d\beta(t)}{dt} + i\frac{d\bar{u}_z}{dt} - \alpha\bar{u}_z^{\ 2}\right) + \sigma\alpha\left(\alpha^2 - \frac{1}{R_0^{\ 2}}\right) = 0$$

Jet Breakup

Dominant wave number



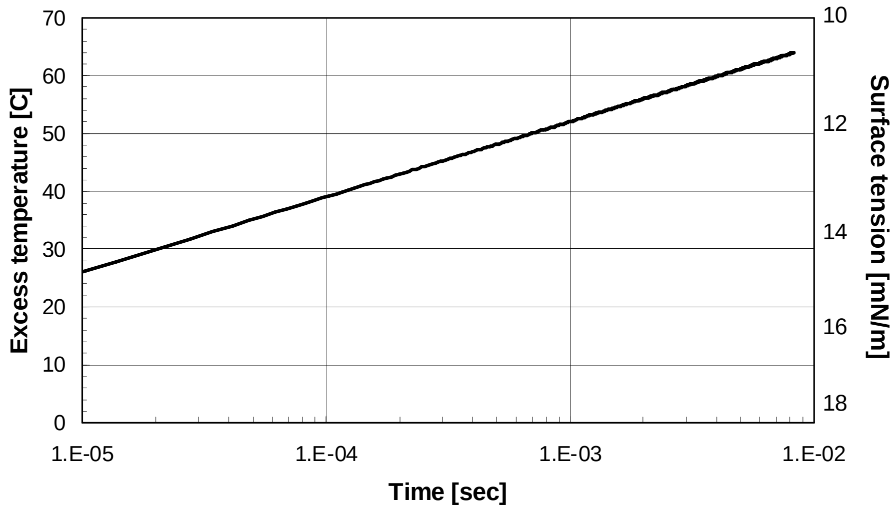
Jet Breakup

Spatial surface Tension variation - Marangoni effect

Typical 10micron droplets break at 0.1ms rate:

$$\tau \approx d\sigma/dx \approx \frac{(18-13)mN/m}{10\mu m} = 0.5kPa$$

$$\tau \approx \sigma/d \approx \frac{13mN/m}{(10\mu m)^2} = 1.3kPa$$

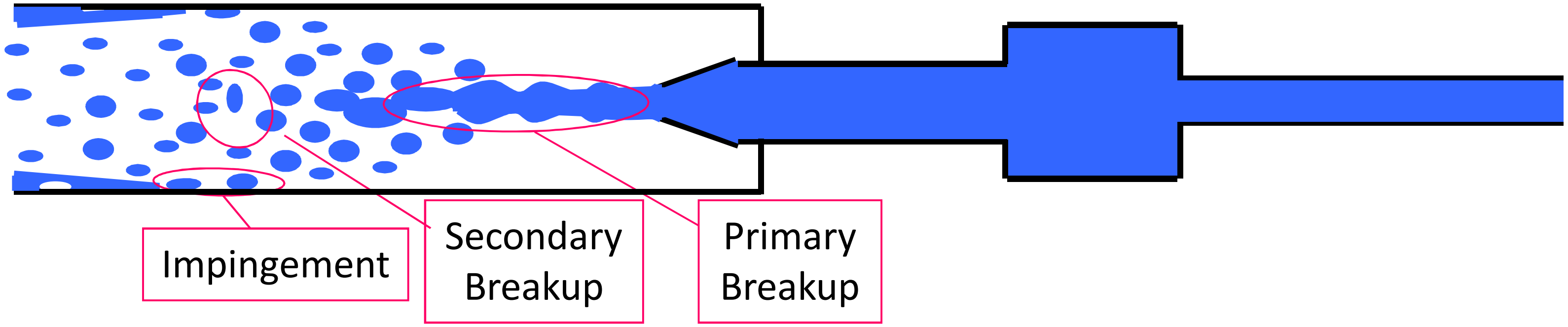


Jet Breakup

An analytic approach to analyse transient effects on jets instabilities:

- Transient jet velocity

- Transient surface tension (injection to hot ambient)?



Thank You!